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Cylindrical self-consistent solutions of semiclassical gravity

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Abstract

The possibility of the creation of curved spacetimes or spacetimes with a nontrivial topological structure by the vacuum fluctuations of quantum fields is discussed. This problem is considered in the framework of the self-consistent semiclassical theory of gravity. The approximate method for obtaining the vacuum expectation value of the renormalized stress–energy tensor of conformally invariant quantum fields in static spacetimes is used. The particular solutions of Einstein equations for the different boundary conditions at the cylinder symmetry axis are obtained. © 1998 Elsevier Science B.V.

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There is a very interesting question: can the vacuum fluctuation of quantum fields be the single source of curvature or nontrivial topological structure of spacetime? One may find the solution of this problem in the theory of quantum gravity. But a fully satisfactory theory of quantum gravity does not yet exist. As a first step on the way to solution of this problem one may use the self-consistent semiclassical theory of gravity. This theory sets the classical Einstein tensor equal to the expectation value of the stress–energy tensor operator of the quantized matter fields present (throughout we use units such that $c = \hbar = G = 1$),

$$G_{\mu\nu} = 8\pi\langle T_{\mu\nu}\rangle. \tag{1}$$

The primary difficulty in the theory of semiclassical gravity is that the effects of the quantized gravitational field are ignored. A secondary difficulty of semiclassical gravity is that $\langle T_{\mu\nu}\rangle$ strongly depends on the metric and is generally difficult to calculate. Most calculations of $\langle T_{\mu\nu}\rangle$ have been made on a fixed classical background. These results cannot be used in the self-consistent semiclassical theory. There have been approximate calculations made for conformally coupled fields in any static spherically symmetric Einstein spacetime [1,2]. The analytical approximations to $\langle T_{\mu\nu}\rangle$ have been obtained by Anderson, Hiscock and Samuel [3] for massless or massive scalar fields with an arbitrary coupling ξ to scalar curvature in static spherically symmetric spacetimes. Khatsymovsky has calculated in Ref. [4] the renormalized stress–energy tensor of an electromagnetic vacuum in static spherically symmetric spacetimes. In this work we use the Killing ansatz giving the approximate vacuum expectation value of the stress–energy tensor $\langle T_{\mu\nu}\rangle$ for conformally coupled massless field in static

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